Physics 523
November 8, 2000

Do problems 5.0 and 5.1.

Problem 5.0: Consider the complex scalar field of Eq.(5.2.11). Compute the mean value in the vacuum of the time-ordered product

$$-i\,\Delta(x,y) = \big(\Phi_0,\, T\,\{\phi(x)\,\phi^\dagger(y)\}\,\Phi_0\big) = \langle 0|\, T\,\{\phi(x)\,\phi^\dagger(y)\}\,|0\rangle \qquad (1)$$

or

$$-i\,\Delta(x,y) = \theta(x^0-y^0)\,\langle 0|\,\phi(x)\,\phi^\dagger(y)\,|0\rangle + \theta(y^0-x^0)\,\langle 0|\,\phi(y^\dagger)\,\phi(x)\,|0\rangle \quad (2)$$

in which all quantities are those of free fields. You may use Eq.(6.2.15) to express $\theta(x^0-y^0)$ as an integral over an energy variable s and then change variables to $q = (\vec{p}, p^0 + s)$ where $p^0 = \sqrt{\vec{p}^2+m^2}$. You may then use the identity

$$\theta(-t) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{-ist}}{s-i\epsilon} ds \qquad (3)$$

to write $\theta(y^0-x^0)$ as an integral over an energy variable s and then change variables to $r = (-\vec{p}, s-p^0)$. In both cases it will behoove you to imitate Weinberg's arguments on pages 276 and 277.

Hints for problem 5.1: You may need to use Eq.(2.5.10). Problem 5.1 is easier in matrix notation. If we suppress the species index n, which plays no role, and introduce the matrix

$$U_{l,\sigma}(\vec{p}) = u_l(\vec{p},\sigma), \qquad (4)$$

then we may write Eq.(5.1.23) as

$$U(\vec{0})\,D^{(j)}(R) = D(R)\,U(\vec{0}). \qquad (5)$$

Eq.(5.1.21) then is

$$U(\vec{q}) = (\frac{m}{q^0})^{1/2}\,D(L(q))\,U(\vec{0}), \qquad (6)$$

and Eq.(5.1.19) is

$$U(\vec{p}_\Lambda)\, D^{(j)}(W(\Lambda,p)) = (\frac{p^0}{(\Lambda p)^0})^{1/2}\, D(\Lambda)\, U(\vec{p}). \qquad (7)$$

In these formulae the matrices D without the superscript (j) are (in general, non-unitary) matrices in a representation of the Lorentz group. The matrices $D^{(j)}(R)$ are the usual unitary matrices that represent the (compact) rotation group.